

Understand Structure

SMP 7

Region 10 Algebra Forum

MaTHink 2013 Mini Conference

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“The important thing is not to stop questioning. Curiosity has its own reason for existing. One cannot help but be in awe when he contemplates the mysteries of eternity, of life, **of the marvelous structure of reality**. It is enough if one tries merely to comprehend a little of this mystery every day. Never lose a holy curiosity.”

Albert Einstein



SMP 7 - Look for and Make Use of Structure

DO STUDENTS:

- **Look closely to determine a pattern or structure?**
- **Recognize and utilize properties?**
- **Decompose and recombine numbers and expressions?**
- **Have the facility to shift perspectives?**

Standards for Mathematical Practice

STRUCTURE ?

- mode of building, construction, or organization; arrangement of parts, elements, or constituents

Example: a pyramidal structure.

- the relationship or organization of the component parts of a whole

Example: the structure of a poem.

Mathematical Structure

Mathematical structure is a description of **properties** that are preserved or maintained under a set of **actions**.

The properties themselves, instantiated in many different contexts, constitute a structure.

John Mason

<http://nrich.maths.org/7472>

Double die or two dice

- Roll the double die.
- One of the numbers obtained is the tens value of a two-digit number and the other one, is the ones.
- Calculate the difference between that number and the one obtained by switching its digits.
- **What do you notice?**



In this case:

$$41 - 14 = 27$$

$$64 - 46 = 18$$

$$61 - 16 = 45$$

$$52 - 25 = 27$$

$$54 - 45 = 9$$

$$33 - 33 = 0$$

$$41 - 14 = 27$$




$$21 - 12 = 9$$

An explanation:

$$66 - 66 = 0$$

 -10


$$66 - 56 = 10$$

 -1

$$65 - 56 = 10 - 1 = 9$$

 -10


$$64 - 46 = 18$$

 -1

$$63 - 36 = 27$$

 -10

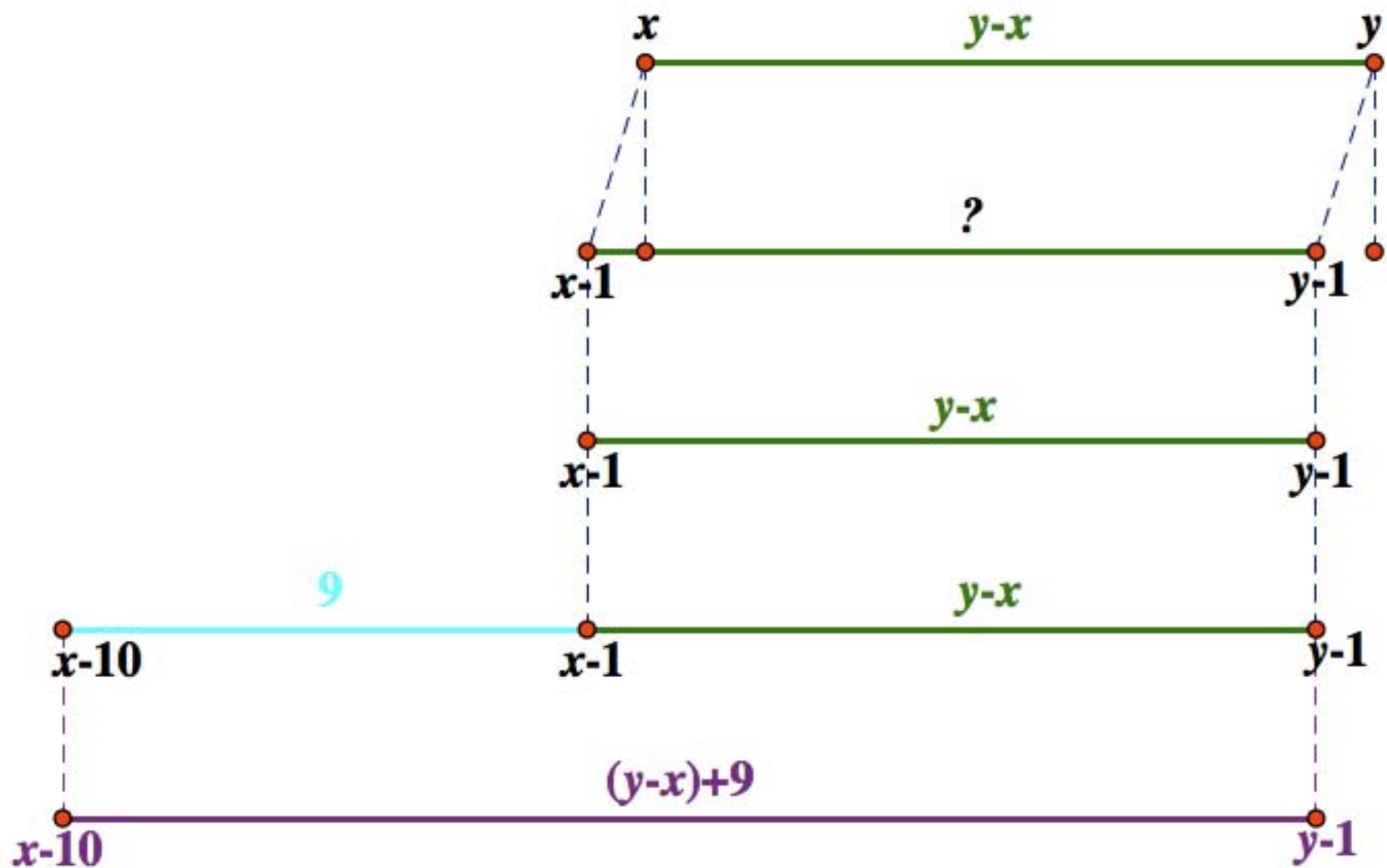
$$62 - 26 = 36$$

 -1

$$61 - 16 = 45$$

 -10

Another explanation:



I took the graph of the line

$$y = 4x + 7$$

and performed the following four transformations:

Reflection across the y-axis

Translation 3 units down

Translation 2 units to the left

Reflection across the x-axis



Unfortunately,

I can't remember the order in which I carried out the four transformations, but I know that I ended up with the graph of the line

$$y = 4x - 2.$$

Find an order in which I could have carried out the transformations.

There is more than one way of doing this.

Can you find them all?

Explain why different orders can lead to the same outcome.

What other lines could I have ended up with
if I had performed these four transformations in a different order?

If I start with a line of equation $y = ax + b$ and use the same four
transformations, what other lines could I have ended up with?

If I start with a line of equation $y = ax + b$ and use the same
reflections and a translation of \underline{u} units to the left and a
translation of \underline{v} units down ,
what other lines could I have ended up with?

Notation:

T_a – Translation with vector $\langle a, 0 \rangle$

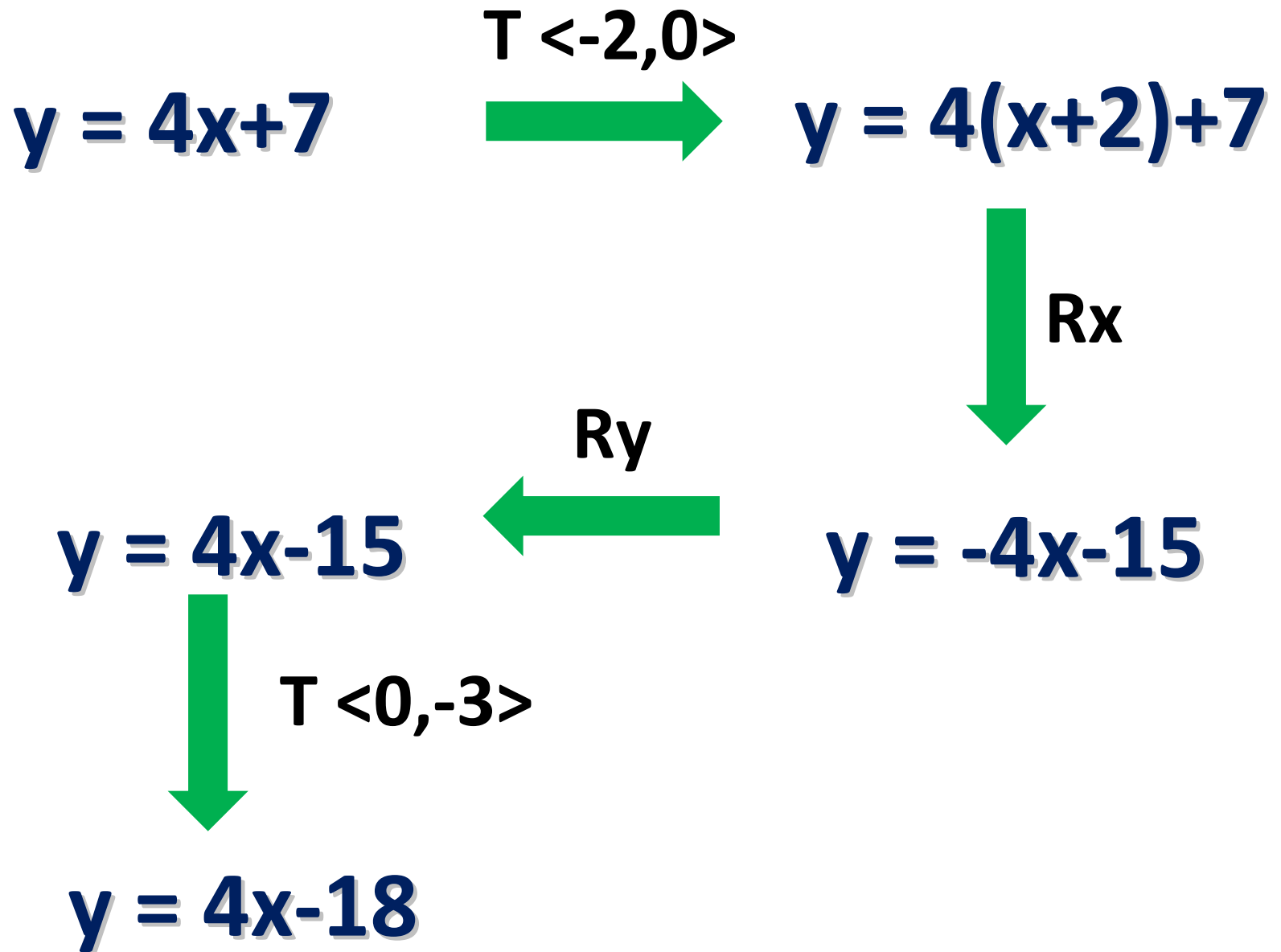
T_b – Translation with vector $\langle 0, b \rangle$

R_x – Reflection across the x-axis

R_y – Reflection across the y-axis

$$y = 4x + 7 \quad \xrightarrow{\quad ? \quad} \quad y = 4x - 2$$

Hint: $4! = 24$



$$R\left(-\frac{a}{2}, -\frac{b}{2}\right), 180^\circ$$

Rx ° Ry ° Ta ° Tb
Ry ° Rx ° Ta ° Tb
Ry ° Ta ° Rx ° Tb
Rx ° Ry ° Tb ° Ta
Rx ° Tb ° Ry ° Ta
Ry ° Rx ° Tb ° Ta

Half-turn around
point (1, 1.5)
In this case
y=4x+7 → y = 4x-12

$$R\left(-\frac{a}{2}, \frac{b}{2}\right), 180^\circ$$

Tb ° Rx ° Ry ° Ta
Tb ° Ry ° Rx ° Ta
Ry ° Tb ° Rx ° Ta
Tb ° Ry ° Ta ° Rx
Ry ° Tb ° Ta ° Rx
Ry ° Tb ° Tb ° Rx

Half-turn around
point (1, -1.5)
In this case
y=4x+7 → y = 4x-18

$$R\left(\frac{a}{2}, -\frac{b}{2}\right), 180^\circ$$

Ta ° Ry ° Rx ° Tb
Ta ° Rx ° Ry ° Tb
Rx ° Ta ° Ry ° Tb
Rx ° Ta ° Tb ° Ry
Rx ° Tb ° Ta ° Ry
Ta ° Rx ° Tb ° Ry

Half-turn around
point (-1, 1.5)
In this case
y=4x+7 → y = 4x+4

$$R\left(\frac{a}{2}, \frac{b}{2}\right), 180^\circ$$

Ta ° Tb ° Rx ° Ry
Tb ° Ta ° Rx ° Ry
Tb ° Rx ° Ta ° Ry
Ta ° Tb ° Ry ° Rx
Tb ° Ta ° Ry ° Rx
Ta ° Rx ° Tb ° Rx

Half-turn around
point (-1, -1.5)
In this case
y=4x+7 → y = 4x-2

Looking for structure must be a guiding principle when trying to make sense of mathematical concepts.



Thank you!